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LOCAL INSTABILITY OF THE WALLS OF BOREHOLES IN DRILLING IN COMPRESSIBLE HARDENING ELASTOVISCOPLASTIC MEDIA

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The local instability of borehole walls in complex compressible media is studied within the framework of exact three-dimensional equations. Numerical experiments were performed for particular materials. The effect of the dilatancy rate, the viscosity, the gravity parameter, and Poisson's ratio on the critical parameters is estimated.

It is well known that the solution of the problems of rock mechanics related to drilling of oil and gas boreholes is reduced to the formulation and solution of the problems of local instability of the rock zone adjacent to the shaft in the presence of elastoplastic deformations [1–4]. This is due to the fact that even at depths smaller than 1 km, the stresses around vertical workings and boreholes exceed the ultimate strength of the rock; this results in inelastic deformation before the local loss of elastic stability occurs. Evidently, to study the problem of stability of a rock working, one should use more complicated models that describe the behavior of rocks most adequately [4]. In this paper, in contrast to [3], the local instability of the rock in the shaft zone is modeled by relations of a compressible elastoviscoplastic body with translational hardening [5–7].

In this case, the loading function is written in the form

$$F = \alpha\sigma + \sqrt{(S_i^j - c(e_i^j)^{p'} - \eta(\dot{e}_i^j)^{p'})(S_i^j - c(e_i^j)^{p'} - \eta(\dot{e}_i^j)^{p'})} - \sqrt{2}K, \tag{1}$$

and the relations of the associated flow law have the form

$$(\dot{e}_i^j)^p = \xi\Big(\frac{\alpha}{3}\, g_i^j + \frac{S_i^j - c(e_i^j)^{p'} - \eta(\dot{e}_i^j)^{p'}}{\sqrt{2}K - \alpha\sigma}\Big). \tag{2}$$

Here α is the dilatancy rate, c is the hardening coefficient, η is the viscosity coefficient, K is the yield point, $(e_i^j)^{p'}$ and $(\dot{e}_i^j)^{p'}$ are the deviatoric plastic-strain and plastic strain-rate tensors, $\sigma = (1/3)\sigma_k^k$, $S_i^j = \sigma_i^j - \sigma\delta_i^j$ is the deviatoric stress tensor, δ_i^j is the Kronecker symbol, g_i^j is the fundamental tensor, and ξ is a positive factor. The subscripts and superscripts i, j, and k run from 1 to 3 and the superscript p' denotes the deviatoric part of the tensor in the plasticity region. Summation over the repeated indices is performed. Relation (2) takes into account the associated compressibility of the material, which is related to the occurrence of plastic shear strains in the body.

A stability analysis of the prebuckling state of a body of volume V which is characterized by the displacement vector $\overset{\circ}{u}_i(x_k,t)$, the stress tensor $\overset{\circ}{\sigma}{}_i^j(x_k,t)$, and the vector of the body $\overset{\circ}{X}_i$ and surface $\overset{\circ}{P}_i$ forces reduces to the solution of a system of differential equations in variations under the corresponding boundary conditions [8].

The equations of equilibrium for the plastic V^p and elastic V^e regions have the form

$$\nabla_i(\sigma_j^i + \overset{\circ}{\sigma}{}_\alpha^i \nabla^\alpha u_j) + X_j - \rho s^2 u_j = 0, \qquad s = i\omega, \tag{3}$$

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where the symbol ∇ denotes covariant differentiation.

The boundary conditions at the outer surface S_p^p (and, hence, S_p^e) are

$$(\sigma_j^i + \overset{\circ}{\sigma}{}^i_\alpha \nabla^\alpha u_j) n_i = p_j, \tag{4}$$

Here we have $p_j = \overset{\circ}{p}_k \nabla^k u_j$ and $X_j = \overset{\circ}{X}_k \nabla^k u_j$ in the case of a "follower" load and $p_j = X_j = 0$ in the case of a "dead" load. Here and henceforth, the superscripts p and e refer to quantities corresponding to the plastic and elastic regions, respectively, and the circle atop refers to the components of the unperturbed prebuckling state.

The relationship between the amplitude values of the stresses and displacements in the plastic and elastic regions can be written in the form

$$\sigma_j^i = a_{i\alpha} g^{\alpha\alpha} \nabla_\alpha u_k g_j^i + (1 - g_j^i) g^{ii} G_j^i (\nabla_i u_j + \nabla_j u_i) \tag{5}$$

(no summation over i and j). In the plastic region, the coefficients $a_{i\alpha}$ and G_j^i have the form

$$a_{i\alpha} = \frac{E}{1+\nu} \delta_{i\alpha} + \overset{\circ}{f}_{\alpha\alpha} B_{ii} + A_{ii}, \qquad G_j^i = \frac{E}{2(1+\nu)} = G \tag{6}$$

(no summation over i and α), where

$$A_{ij} = \frac{E}{1+\nu}\Big(\frac{1}{3}\delta_{ij} - \frac{AB}{(a\alpha)^2 D}\lambda_{ij} - \frac{\overset{\circ}{f}_{ij}}{a\alpha}\Big), \quad B_{ij} = \frac{E}{1+\nu}\Big(\frac{\overset{\circ}{E}B}{Da\alpha} + \frac{C}{a\alpha}\Big)\lambda_{ij}, \quad a = \sqrt{2}K - \alpha\overset{\circ}{\sigma},$$

$$\lambda_{ij} = \frac{3\overset{\circ}{f}_{ij}(2\nu - 1)}{a\alpha E} - \frac{\nu+1}{E}\delta_{ij}, \quad A = \overset{\circ}{f}_{ij}\overset{\circ}{f}_{ij}, \quad B = 1 + \frac{1+\nu}{E}C, \quad C = c + s\eta,$$

$$D = \frac{1+\nu}{E} - \frac{3(2\nu-1)BA}{(a\alpha)^2 E}, \quad \overset{\circ}{E} = 1 - \frac{3AC(2\nu-1)}{(a\alpha)^2 E}, \quad \overset{\circ}{f}_{ij} = \overset{\circ}{S}_{ij} - c\overset{\circ}{e}{}^{p\prime}_{ij}.$$

In the elastic region, the coefficients $a_{i\alpha}$ and G_j^i are determined by relations (6) for $A_{ij} = B_{ij} \equiv 0$, i.e.,

$$a_{i\alpha} = (\lambda + 2\mu) g_{i\alpha}, \qquad G_j^i = \mu. \tag{7}$$

In Eqs. (6) and (7), λ and μ are the Lamé parameters, E is Young's modulus, and ν is Poisson's ratio. We note that the representation (5) is possible only if the prebuckling state is uniform or depends on one variable.

The continuity conditions at the elastoplastic boundary Γ have the form

$$[(\sigma_j^i + \overset{\circ}{\sigma}{}^i_\alpha \nabla^\alpha u_j) n_j] = 0, \qquad [u_i] = 0. \tag{8}$$

Equations (3)–(8) form a closed system of equations for analysis of stability problems where a boundary exists between the regions of elastic and plastic behavior of the material during loading.

Let a round borehole drilled vertically in rock be filled with a liquid of density γ_* and its walls be subjected to the pressure $q = \gamma_* h$, where h is the depth.

The pressure q is called the backpressure of a drilling fluid that prevents the change in shape and dimensions of the cross section of the boreholes. We model [9] the rock mass with a borehole by a weightless infinite plane with a round hole of radius a whose contour is loaded by the uniformly distributed pressure q. At infinity, the stresses in the plate tend to $\gamma_n h$, where γ_n is the density of the rock. The stress distribution in the unperturbed rock mass is assumed to be hydrostatic: $p = \gamma_n h$.

In determining the stress and strain components in the prebuckling state in the axisymmetric case, all the functions are written in the form of series in powers of the parameter α, i.e., the dilatancy rate: $\{\sigma_{ij}, e_{ij}^p, e_{ij}^e, \xi, \ldots\} = \sum_{n=0}^{\infty} \alpha^n \{\sigma_{ij}^{(n)}, e_{ij}^{p(n)}, e_{ij}^{e(n)}, \xi^{(n)}, \ldots\}$.

The zeroth approximation corresponds to the incompressible elastoviscoplastic medium in the region V^p and has the form [8]:

— in the plastic region ($a^* < r < 1$),

$$\sigma_r^{(0)} = -q - \frac{\chi}{c+2G}\Big(4G\ln\frac{r}{a^*} + \frac{c}{a^{*2}} - \frac{c}{r^2}\Big), \qquad e_\theta^{p(0)} = -\frac{\chi}{c+2G}\Big(\frac{1}{r^2} - 1\Big),$$

$$\sigma_\theta^{(0)} = -q - \frac{\chi}{c+2G}\Big(4G + 4G\ln\frac{r}{a^*} + \frac{c}{a^{*2}} + \frac{c}{r^2}\Big), \quad u^{(0)} = \frac{\chi}{2Gr}, \quad \chi = \operatorname{sign}(q-p);$$

— in the elastic region ($1 < r < \infty$),

$$\sigma_r^{(0)} = -p + \frac{\chi}{r^2}, \qquad \sigma_\theta^{(0)} = -p - \frac{\chi}{r^2}, \qquad u^{(0)} = \frac{\chi}{2Gr}.$$

Here the quantities having the dimension of stress are divided by $k = \sqrt{2}K/2$, whereas the quantities having the dimension of length are divided by the radius of the elastoplastic boundary in the zeroth approximation $r_s^{(0)}$. The quantity $a^* = a/r_s^{(0)}$ is determined from the equation $|q-p|(c+2G) - 2G + 4G\ln a^* - c/a^{*2} = 0$.

We write the first approximation in the form of the following relations:

— in the plastic region ($a^* < r < 1$),

$$\sigma_r^{(1)} = C_2 + \tilde{B}\ln r + \frac{\tilde{C}}{2r^2} + \tilde{D}(\ln r)^2 - \tilde{E}\Big(\frac{\ln r}{2r^2} + \frac{1}{4r^2}\Big),$$

$$\sigma_\theta^{(1)} = C_2 + \tilde{B}\ln r + \tilde{B} + \frac{\tilde{C}}{2r^2} + \tilde{D}\ln r\Big(\frac{1}{2}\ln r + 1\Big) - \tilde{E}\Big(\frac{1}{4r^2} - \frac{\ln r}{2r^2} + \frac{1}{4r^2}\Big), \tag{9}$$

$$u^{(1)} = \frac{1}{c+2G}\Big(\frac{\ln r}{r} - \frac{r}{2}\Big) + \frac{C_1}{r};$$

— in the elastic region ($1 < r < \infty$),

$$\sigma_r^{(1)} = \frac{C_3}{r^2}, \qquad \sigma_\theta^{(1)} = -\frac{C_3}{r^2}, \qquad u^{(1)} = \frac{C_3}{2Gr}, \tag{10}$$

where

$$\tilde{B} = 2qA + A^2 + \frac{A^2c}{2a^{*2}} - 2A^2\ln a^*, \quad \tilde{C} = C_1Ac - \frac{Ac}{2c+4G}, \quad \tilde{D} = 2A^2, \quad \tilde{E} = \frac{Ac}{2c+4G},$$

$$C_1 = \Big(2G + Ac\,\frac{1-a^{*2}}{2a^{*2}}\Big)^{-1}\Big(\frac{G}{c+2G} + \frac{Ac(1-a^{*2})}{2a^{*2}(c+2G)} + \tilde{B}\ln a^* - \tilde{E}\Big(\frac{\ln a^*}{2a^{*2}} - \frac{1}{4a^*} + \frac{1}{4}\Big) + \frac{\tilde{D}}{2}(\ln a^*)^2\Big),$$

$$C_2 = -\frac{\tilde{C}}{2a^{*2}} - \tilde{B}\ln a^* + \tilde{E}\Big(\frac{\ln a^*}{2a^{*2}} + \frac{1}{4a^*}\Big) - \frac{\tilde{D}}{2}(\ln a^*)^2, \quad C_3 = \frac{G}{c+2G} - 2GC_1.$$

In the first approximation, the equation for the radius r_{s1} of the plastic boundary has the form

$$r_{s1} = (\sigma_\theta^{(1)e} - \sigma_\theta^{(1)p})\Big/\Big(\frac{d\sigma_\theta^{(1)p}}{dr} - \frac{d\sigma_\theta^{(1)e}}{dr}\Big)\Big|_{r=1} = -\frac{\sigma_\theta^{(1)e} - \sigma_\theta^{(1)p}}{8G}(c+2G)\Big|_{r=1}. \tag{11}$$

In determining the zeroth and first approximations, we used the equations of equilibrium, the plasticity condition (1), the relations of the associated law of plastic flow (2), the relations between total, elastic, and plastic strains, the general equations of the theory of elasticity, the boundary conditions, and the conjugation conditions for the solutions in the elastic and plastic regions.

In accordance with (4), the boundary conditions on the borehole surface and the conditions of perturbation decay as $r \to \infty$ are written in the form

$$\Big(a_{11}u_{,r} + a_{12}\frac{1}{r}v_{,\theta} + a_{12}\frac{1}{r}u + a_{13}w_{,z}\Big)\Big|_{r=a} = 0, \quad (rv_{,r} + u_{,\theta} - v)\Big|_{r=a} = 0, \quad (w_{,r} + u_{,z})\Big|_{r=a} = 0,$$

$$u \underset{r\to\infty}{\longrightarrow} 0, \qquad v \underset{r\to\infty}{\longrightarrow} 0, \qquad w \underset{r\to\infty}{\longrightarrow} 0.$$

According to (5), for the amplitudes of displacement and stress perturbations, the conjugation conditions (8) at the elastoplastic boundary r_s take the form

$$[u] = 0, \quad [v] = 0, \quad [w] = 0, \qquad \Big[a_{12}\,\frac{1}{r}\,u + \big(a_{11} + \overset{\circ}{\sigma}{}_1^1\big)u_{,r} + a_{12}\,\frac{1}{r}\,v_{,\theta} + a_{13}w_{,z}\Big] = 0,$$
$$\Big[G_3^1 u_{,z} + \big(G_3^1 + \overset{\circ}{\sigma}{}_1^1\big)w_{,r}\Big] = 0, \quad \Big[G_2^1 u_{,\theta} + \big(G_2^1 + 2\overset{\circ}{\sigma}{}_1^1\big)v + \big(G_2^1 + \overset{\circ}{\sigma}{}_1^1\big)rv_{,r}\Big] = 0. \tag{12}$$

In the quasistatic formulation with allowance for (5), Eqs. (3) are reduced to the following system of partial differential equations with variable coefficients:

$$u\Big(\frac{1}{r}\,a_{12,1} - \frac{1}{r^2}\big(a_{22} + \overset{\circ}{\sigma}_\theta\big)\Big) + u_{,r}\Big(a_{11,1} - \frac{1}{r}\big(a_{12} + a_{11} - a_{21} + \overset{\circ}{\sigma}_\theta\big)\Big) + u_{,rr}(a_{11} + \overset{\circ}{\sigma}_r)$$
$$+ u_{,\theta\theta}\,\frac{1}{r^2}\,(G_1^2 + \overset{\circ}{\sigma}_\theta) + u_{,zz}(G_1^3 + \overset{\circ}{\sigma}_z) + v_{,\theta}\Big(\frac{1}{r}\,a_{12,1} - \frac{1}{r^2}\,(G_1^2 a_{22} + 2\overset{\circ}{\sigma}_\theta)\Big)$$
$$+ v_{,r\theta}\,\frac{1}{r}\,(G_1^2 + a_{12}) + w_{,z}\Big(a_{13,1} + \frac{1}{r}\,(a_{13} - a_{23})\Big) + w_{,rz}(G_3^1 + a_{13}) = 0,$$
$$u_{,\theta}\Big(G_{2,1}^1 + \frac{1}{r}\,(a_{22} + G_1^2 + 2\overset{\circ}{\sigma}_\theta)\Big) + u_{,r\theta}(a_{21} + G_2^1)$$
$$+ v\Big(- G_{2,1}^1 + \frac{1}{r}\,(-G_1^2 + \overset{\circ}{\sigma}_\theta)\Big) + v_{,r}r(G_{2,1}^1 + G_1^2 + \overset{\circ}{\sigma}_\theta) + v_{,rr}r(G_2^1 + \overset{\circ}{\sigma}_z) \tag{13}$$
$$+ v_{,\theta\theta}\,\frac{1}{r}\,(a_{22} + \overset{\circ}{\sigma}_\theta) + v_{,zz}r(G_2^3 + \overset{\circ}{\sigma}_z) + w_{,\theta z}(G_2^3 + a_{23}) = 0,$$
$$u_{,z}\Big(G_{3,1}^1 + \frac{1}{r}\,(a_{32} + G_3^1)\Big) + u_{,rz}(a_{31} + G_3^1) + v_{,\theta z}\,\frac{1}{r}\,(a_{32} + G_3^2)$$
$$+w_{,r}\Big(G_{3,1}^1 + \frac{1}{r}\,(G_3^1 + \overset{\circ}{\sigma}_\theta)\Big) + w_{,rr}(G_3^1 + \overset{\circ}{\sigma}_r) + w_{,\theta\theta}\,\frac{1}{r^2}(G_3^2 + \overset{\circ}{\sigma}_\theta) + w_{,zz}(a_{33} + \overset{\circ}{\sigma}_z) = 0.$$

In the elastic region, the equations of equilibrium (13) are also satisfied, where it is necessary to set $A_{ij} = B_{ij} \equiv 0$ for the coefficients $a_{i\alpha}$ found from (6), as was pointed out above.

We seek a solution of Eqs. (13) in the form $u = A^p(r)\cos(m\theta)\cos(nz)$, $v = B^p(r)\sin(m\theta)\cos(nz)$, and $w = C^p(r)\cos(m\theta)\sin(nz)$, where m and n are the wavenumbers.

Using the functions A^p, B^p, and C^p, we write system (13) in the form

$$A^p\xi_1 + A'^p\xi_2 + A''^p\xi_3 + B^p\xi_4 + B'^p\xi_5 + C^p\xi_6 + C'^p\xi_7 = 0,$$
$$A^p\xi_8 + A'^p\xi_9 + B^p\xi_{10} + B'^p\xi_{11} + B''^p\xi_{12} + C^p\xi_{13} = 0, \tag{14}$$
$$A^p\xi_{14} + A'^p\xi_{15} + B^p\xi_{16} + C^p\xi_{17} + C'^p\xi_{18} + C''^p\xi_{19} = 0.$$

Here the primes at the functions $A(r)$, $B(r)$, and $C(r)$ denote differentiation with respect to r.

For the region V^e, the functions A^p, B^p, and C^p in system (14) should be replaced by A^e, B^e, and C^e, respectively.

The boundary conditions for $r = a$ and the conditions of perturbation decay as $r \to \infty$ become

$$A^p a_{12}\,\frac{1}{r} + A'^p a_{11} + B^p m\,\frac{1}{r}\,a_{12} + C^p n a_{13} = 0, \quad A^p m + B^p - B'^p r = 0, \quad A^p n - C'^p = 0,$$
$$A^e(r) \to 0, \qquad B^e(r) \to 0, \qquad C^e(r) \to 0. \tag{15}$$

The conjugation conditions (12) at the elastoplastic boundary r_s have the form

$$A\,\frac{a_{12}}{r} + A'(a_{11} + \overset{\circ}{\sigma}{}_1^1) + B\,\frac{m a_{12}}{r} + Cna_{13} = 0,$$
$$-AmG_2^1 + B(2\overset{\circ}{\sigma}{}_1^1 - G_2^1) + B'r(G_2^1 + \overset{\circ}{\sigma}{}_1^1) = 0, \quad -AnG_3^1 + C'(G_3^1 + \overset{\circ}{\sigma}{}_1^1) = 0. \tag{16}$$

In relations (14)–(16), the following notation is introduced:

$$
\begin{gathered}
\xi_1 = \frac{a_{12,1}}{r} - \frac{a_{22} + \mathring{\sigma}_2^2}{r^2} - \frac{m^2}{r^2}\,(G_1^2 + \mathring{\sigma}_2^2) - n^2(G_1^3 + \mathring{\sigma}_3^3), \quad \xi_2 = a_{11,1} + \frac{a_{12} + a_{11} - a_{21} + \mathring{\sigma}_2^2}{r}, \\
\xi_3 = a_{11} + \mathring{\sigma}_1^1, \quad \xi_4 = m\Big(\frac{a_{12,1}}{r} - \frac{G_1^2 + a_{22} + 2\mathring{\sigma}_2^2}{r^2}\Big), \quad \xi_5 = \frac{m}{r}\,(G_1^2 + a_{12}), \\
\xi_6 = n\Big(a_{13,1} + \frac{a_{13} - a_{23}}{r}\Big), \quad \xi_7 = n(G_3^1 + a_{13}), \quad \xi_8 = m\Big(G_{2,1}^1 + \frac{G_1^2 + a_{22} + 2\mathring{\sigma}_2^2}{r}\Big), \\
\xi_9 = -m(G_1^2 + a_{21}), \quad \xi_{10} = -\Big(G_{2,1}^1 + \frac{G_1^2 + \mathring{\sigma}_2^2}{r} + m^2\,\frac{a_{22} + \mathring{\sigma}_2^2}{r} + n^2 r(G_2^3 + \mathring{\sigma}_3^3)\Big), \qquad (17) \\
\xi_{11} = rG_{2,1}^1 + G_1^2 + \mathring{\sigma}_2^2, \quad \xi_{12} = r(G_2^1 + \mathring{\sigma}_2^2), \quad \xi_{13} = -mn(a_{23} + G_2^3), \\
\xi_{14} = -n\Big(G_{3,1}^1 + \frac{G_3^1 + a_{32}}{r}\Big), \quad \xi_{15} = -n(G_3^1 + a_{31}), \quad \xi_{16} = \frac{-mn}{r}\,(G_3^2 + a_{32}), \\
\xi_{17} = \frac{-m^2}{r}\,(\mathring{\sigma}_2^2 + G_3^2) - n^2(\mathring{\sigma}_3^3 + a_{23}), \quad \xi_{18} = G_{3,1}^1 + \frac{G_3^1 + \mathring{\sigma}_2^2}{r}, \quad \xi_{19} = G_3^1 + \mathring{\sigma}_1^1.
\end{gathered}
$$

For the coefficients ξ_i $(i = 1, \ldots, 17)$, the parameters $\mathring{\sigma}_r$, $\mathring{\sigma}_\theta$, and $\mathring{\sigma}_z$ in the region V^p are determined by formulas (9) and a_{ij} and G_j^i by (6) and those in the region V^e are determined by formulas (10) and (7), respectively.

Since we did not succeed in finding an exact analytical solution of the boundary-value problem (14)–(17), we shall seek an approximate solution by the finite-difference method. In this method, the derivatives of the functions $A(r)$, $B(r)$, and $C(r)$ are replaced by finite differences. As a result, we have a homogeneous system of linear algebraic equations, which can be written in matrix form: $\{X_{ij}\}\{Y_i\} = \{0\}$. It follows that the critical parameters (loads) are determined from the solution of the matrix equation

$$
\det\{X_{ij}\} = 0. \qquad (18)
$$

In calculating the determinant, it is necessary to take into account not only the prebuckling stress-strain state [see Eqs. (9) and (10)], but also Eq. (11), which determines the elastoplastic boundary. Minimization should be performed with respect to the difference-grid size h, the wavenumbers over the contour m and along the generatrix n, the parameters of the material and structure λ_j, and the quantity s. Thus, we arrive at the problem of multidimensional optimization of the quantity p depending on s, m, and n under the condition that $F(p, s, m, n, \lambda_j) = 0$.

The problem of determining the critical load for fixed h and λ_j was solved in two stages. At the first stage, the region in the space of the parameters p, s, m, and n, in which the sign of the function $F(p, s, m, n, h, \lambda_j)$ changes, was determined for $0 \leqslant s < \infty$, $0 < p < 1$, and m, $n = 1$, 2, etc. At the second stage, the value of $p_* = \min\limits_{s,m,n} p(s, m, n)$ such that $F(p_*, s_*, m_*, n_*, h, \lambda_j) = 0$ was calculated. Optimization with respect to the parameter h was performed as follows. The calculation of the critical load is terminated if after a twofold decrease in the grid size, the difference between the resulting values of the load corresponds to the specified accuracy. The results of the numerical experiments are given in Figs. 1–3.

Figure 1 shows the critical pressure p_* as a function of the gravity parameter q_* for $E/k = 100$, $\eta_0 = 0.1$, and $c_0 = 0.1$. Curves 1–3 refer to $\nu = 0.3$, curves $1'$–$3'$ to $\nu = 0.5$, curves 1 and $1'$ to $\alpha = 0.1$, curves 2 and $2'$ to $\alpha = 0.4$, and curves 3 and $3'$ to $\alpha = 0.8$. The value of the critical pressure increases as the gravity parameter is increased and it increases significantly (by 10–15%) as Poisson's ratio is increased.

Figure 2 shows the critical value of the gravity parameter q_* for a free working (the value of the load at infinity at which the free working loses stability) versus the dilatancy rate α for $E/k = 100$, $c_0 = 0.1$, and $\nu = 0.3$ (curve 1 and 2 refer to $\eta_0 = 0.1$ and 0.001, respectively). It is clear from Fig. 2 that the critical load decreases as the viscosity increases and, in this sense, the viscosity can be regarded as a destabilizing factor in the medium.

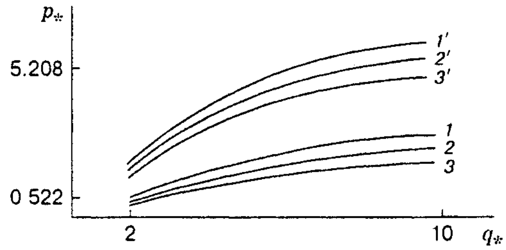

Fig. 1

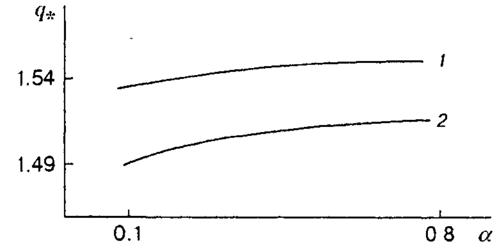

Fig. 2

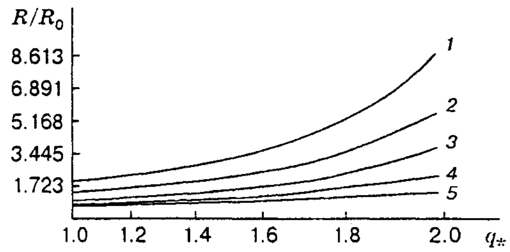

Fig. 3

Figure 3 shows the ratio of the radius of the elastoplastic boundary R to the radius of the working R_0 versus the gravity parameter q_* for a free working. Curves 1–5 refer to $\alpha = 0.70$, 0.55, 0.40, 0.25, and 0.10, respectively, for $E/k = 100$, $\eta_0 = 0.1$, $c_0 = 0.1$, and $\nu = 0.3$. One can see that the radius of the elastoplastic boundary increases with increase in the gravity parameter and dilatancy rate. The curves in Figs. 1–3 correspond to dimensionless values of the mechanical parameters of the materials whose properties are similar to those of allegerit and coal.

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